

**Application of Compressive Sensing to Gravitational
Microlensing Data
- and -
Implications for Miniaturized Space Observatories**

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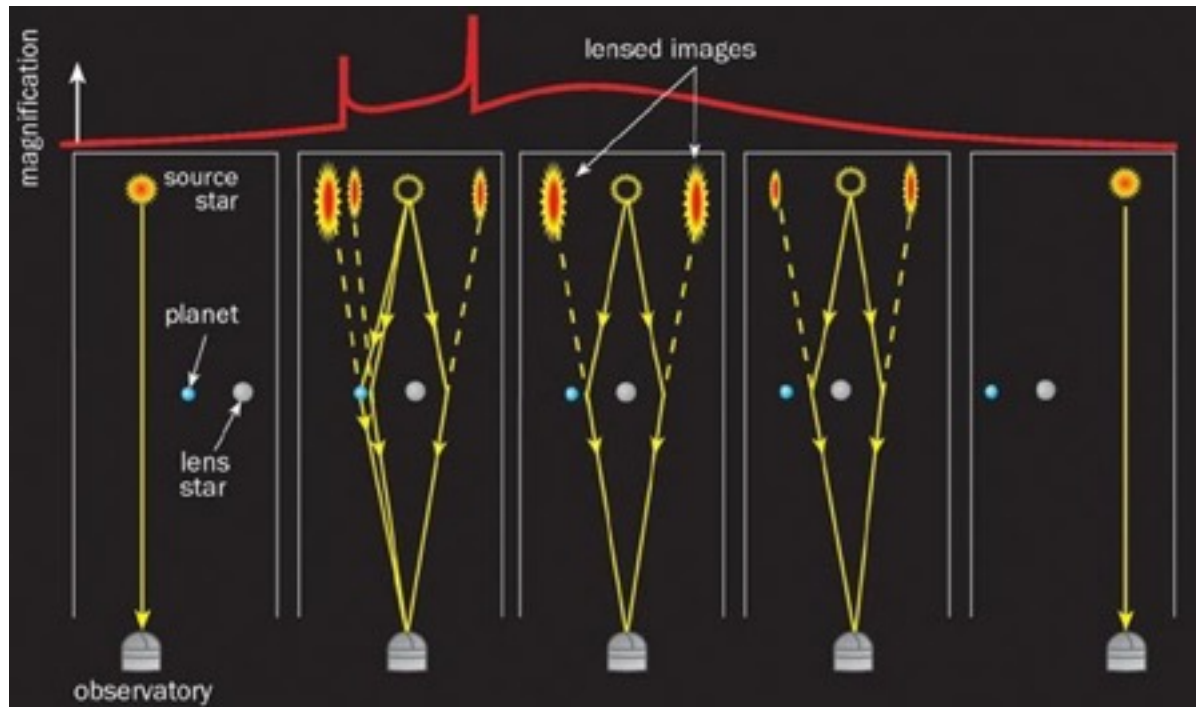


Outline

- Gravitational Microlensing
- Compressive Sensing (CS) Motivation
- Compressive Sensing (CS) Theory
- Single Lens Microlensing Events
- Simulation Results
- Conclusion and Future work

Gravitational Microlensing

- Technique to detect exoplanets and other astrophysical entities



Credit: Space Telescope Science Institute



Current Techniques Limitations

- High rate sampling required to acquire the desired resolution
 - Miniaturized space observatories: Data bandwidth limitation
- Need high cadence for acquiring each image
 - If high cadence is not achieved, an exoplanet transition with a short period can be missed
- Miniaturized space observatories have power and on-board memory limitation
- **How do we achieve high resolution images at a high cadence by acquiring only a few samples?**

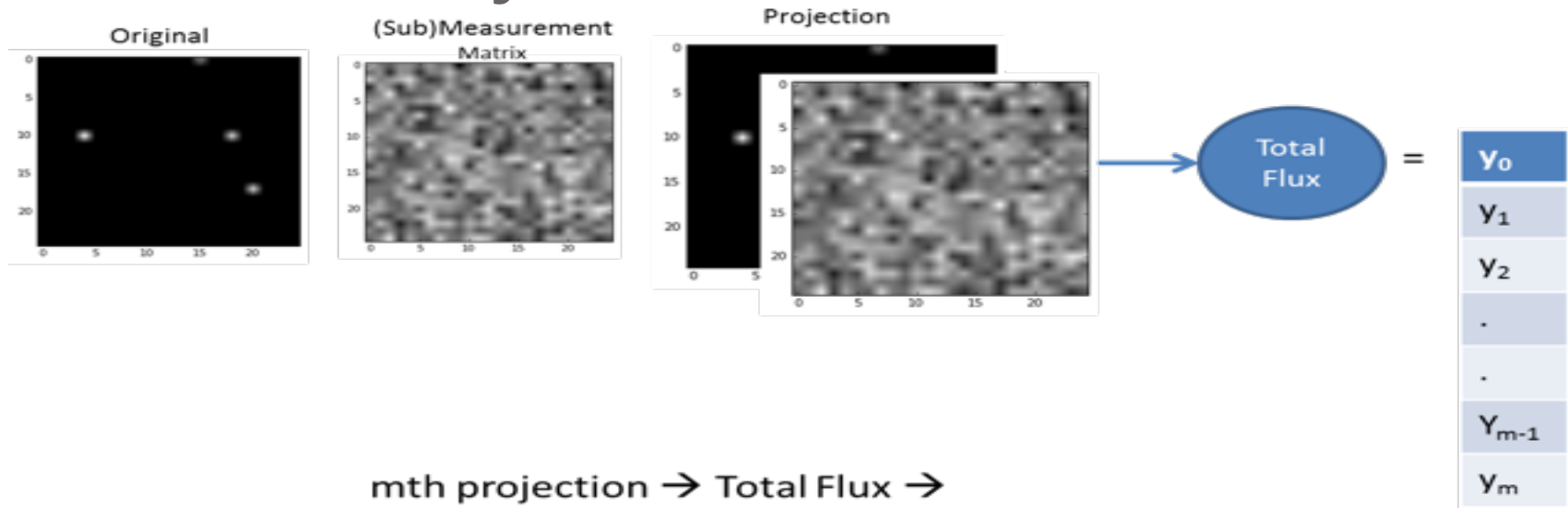


Compressive Sensing (CS) Motivation

- Acquiring each image pixel individually (sampling at the Nyquist rate) is wasteful when the information can be encoded in only a select few samples due to its sparse nature
- Exploit sparsity in images
- Microlensing Events are sparse in spatial domain when differenced
 - That is, at any given time only the stars exhibiting a microlensing event vary in flux
 - Only those stars are evident when differenced with a reference image



CS Theory

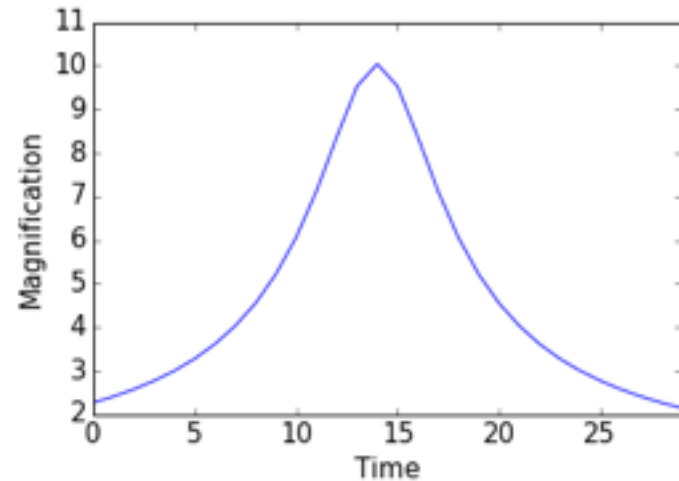
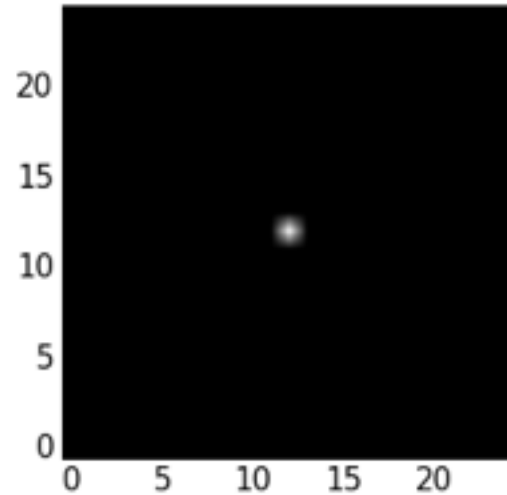


- Each sub measurement matrix gets transformed into a 1D signal representing a row in the measurement matrix.
 - M sub measurement matrices
- Reconstruct original image, given y vector and the associated (sub) measurement matrix for each element in y
 - $Y_{m \times 1} = \Phi_{m \times n} X_{n \times 1}$
 - Optimization (L1 minimization) and greedy algorithms
- A unique solution is obtained only if the original image is sparse in some domain



Single Lens Microlensing Events

- Source star magnification only due to lensing star
- Magnification at each time is dependent on:
 - u_0 : lens-source separation in terms of Einstein's ring radius
 - t_0 : peak magnification time
 - t_e : Einstein's ring radius crossing time



Top: Original spatial domain image at time, $t = 0$



Simulation Setup

All Simulations are performed in **Python**

Gravitational Microlensing Parameters

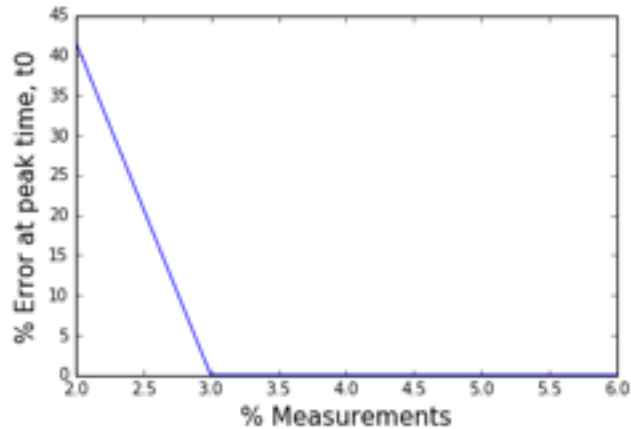
- Single lens event
- $u_0 = 0.1$
- Total 30 time samples
 - Peak magnification at time value = 14
 - Einstein's ring crossing time at time value = 29

CS Parameters

- Image size = 25x25
 - $N = 25 \times 25 = 625$ pixels
- Measurements, M , is varied from 2% of N to 6% of N
 - % Measurements = $\frac{M}{N} \times 100$
- Sparsity: number of non-zero (or significant value) pixels = 1
- Measurement matrix, $:$ Bernoulli Random with 0's and 1's
 - 100 Monte Carlo simulations to vary measurement matrix each time



CS Reconstruction

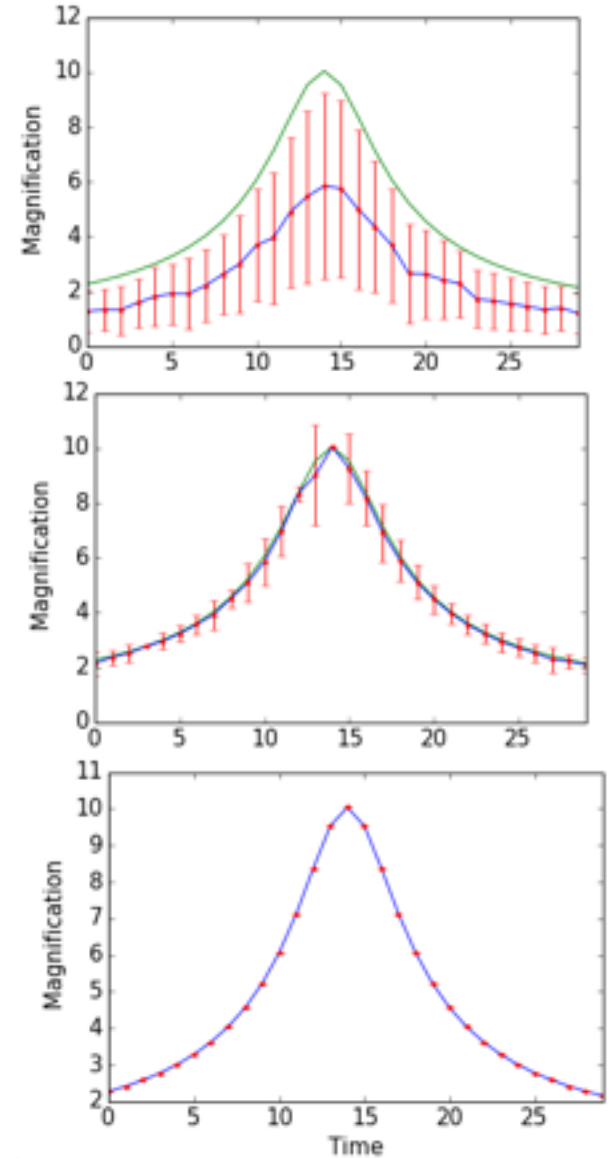


% Error at t0 over center pixel with
3 pixel radius

$$\% \text{ Measurements} = \frac{\# \text{ of measurements}}{\# \text{ of total pixels}} \times 100$$

Green: Original signal
Blue: Reconstructed signal
Red: Error bars

Top: 2% measurements
Middle: 3% measurements
Bottom: 4% measurements





Resolution Accuracy

| % Measurements $\frac{N}{N} \times 100$ | Error Difference in Reconstruction at t_0 | Average Standard deviation over all t | Error Difference in Reconstruction at t_0 | Average Standard deviation over all t |
|---|---|---|---|---|
| 2 | 4.19 | 1.6 | 4.19 | 1.6 |
| 3 | 0.00009 | 0.52 | 0.00009 | 0.52 |
| 4 | 0.00013 | 0.00096 | 0.00013 | 0.00096 |
| 5 | 0.00013 | 0.00078 | 0.00013 | 0.00078 |
| 6 | 0.00016 | 0.00073 | 0.00016 | 0.00073 |

- Change in magnification at peak time, t_0 , is 0.5 units of flux
 - **Resolution error $\ll 0.5$ to capture changes in microlensing curve**
- **4% of N measurements gives optimal error**, along with a low standard deviation, providing lower uncertainty



Conclusions and Future Work

- For a clean image, with very low sparsity, only 4% of Nyquist rate samples are required to reconstruct the image
 - **Significant reduction in data volume and power**
 - **Greatly benefit space flight observatories**
- Future work will include studying
 - Point spread functions and its implications for CS
 - Dense, crowded field images
 - Difference imaging for CS applications
 - Binary lens systems



Acknowledgement

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References

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- [3] S. Seager and R. Dotson, *Exoplanets*. Tucson: University of Arizona Press, 2010.