Solar Image Reconstruction from Visibilities with Compressed Sensing

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X-ray indirect imaging telescopes observing solar flares include the NASA spacecraft RHESSI and its successor STIX on ESA's Solar Orbiter. They measure complex Fourier components of the spatial source distribution (visibilities) as data products. Existing reconstruction algorithms that turn visibilities into images often produce noisy results and require X-ray source-specific parameter tuning. To avoid these issues, we propose to use Compressed Sensing (CS) theory, widely employed in radio-astronomy. This results in a new image reconstruction algorithm for X-ray data based on a simple idea:

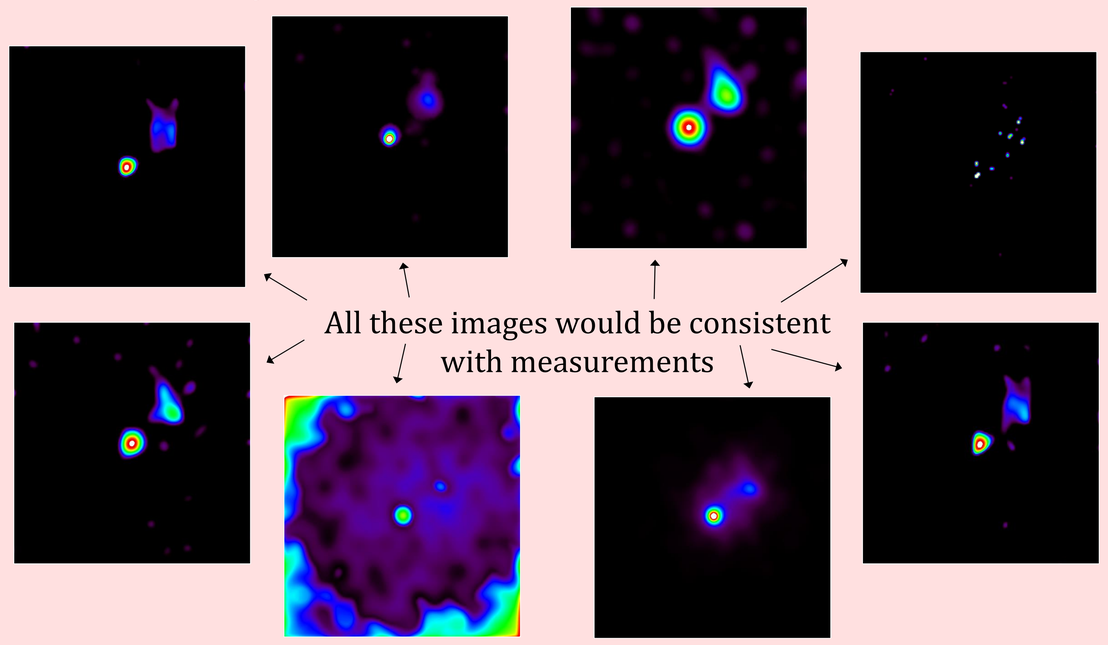
«Find a *plausible* image
consistent with the measurements»

What does consistent mean?

1. Minimize the difference of measured visibilities and visibilities of the reconstructed image
2. Total flux of the reconstructed image must match the total unmodulated X-ray source flux

Consistency itself is not sufficient

There are many potential reconstructions that are consistent with the measurements, because image reconstruction is mathematically an under-determined problem.

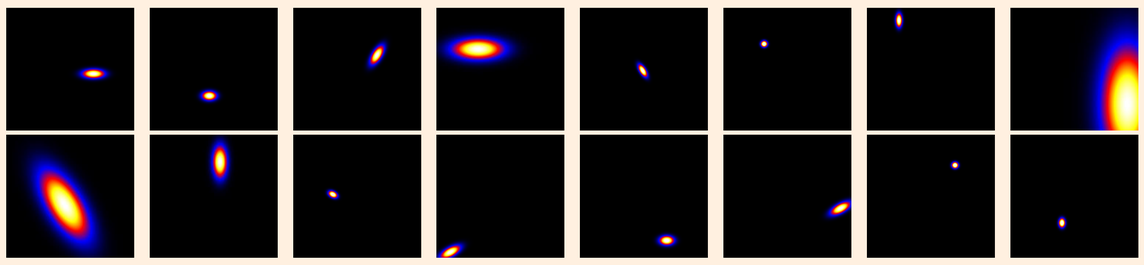


What does plausible mean?

1. Reconstructed image must be positive everywhere
2. According to Compressed Sensing theory:
   "Signals are linear combinations of few basis functions"

$$x = D\alpha$$
$$\Pr(x) = F(\|\alpha\|_0)$$

Candès et al[1] showed that most signals can be reconstructed from limited and noisy samples under these assumptions. For our work, we assume that solar flares can be represented by a linear combination of a few scaled and rotated Gaussians, like these:



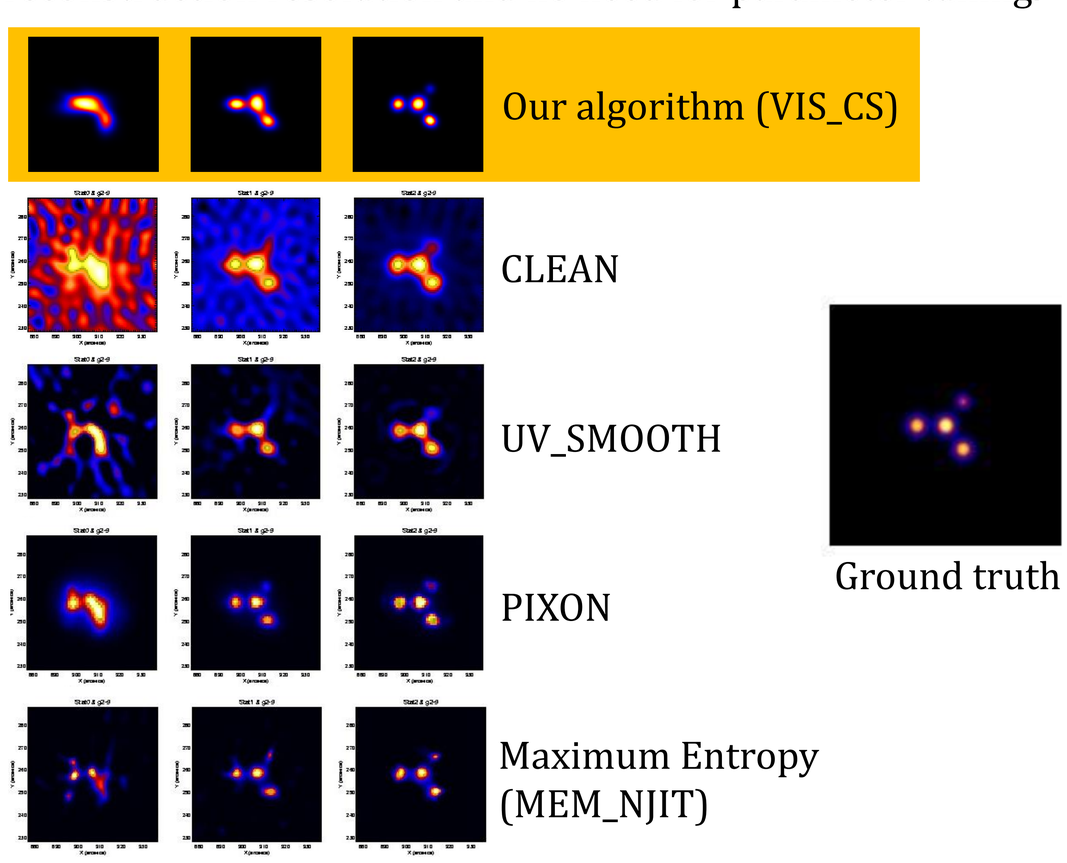
A mathematical model of both objectives

$$\underset{\alpha}{\operatorname{argmin}} \left(\|\alpha\|_1 + \lambda \left\| AD\alpha - \tilde{b} \right\|_2 \right)$$
$$\text{s.t. } \|D\alpha\|_1 = \text{Measured flux}$$
$$\forall \alpha_i \geq 0$$

i.e. find a plausible representation, consistent with the measurements. We solve this model with a QCP solver[2].

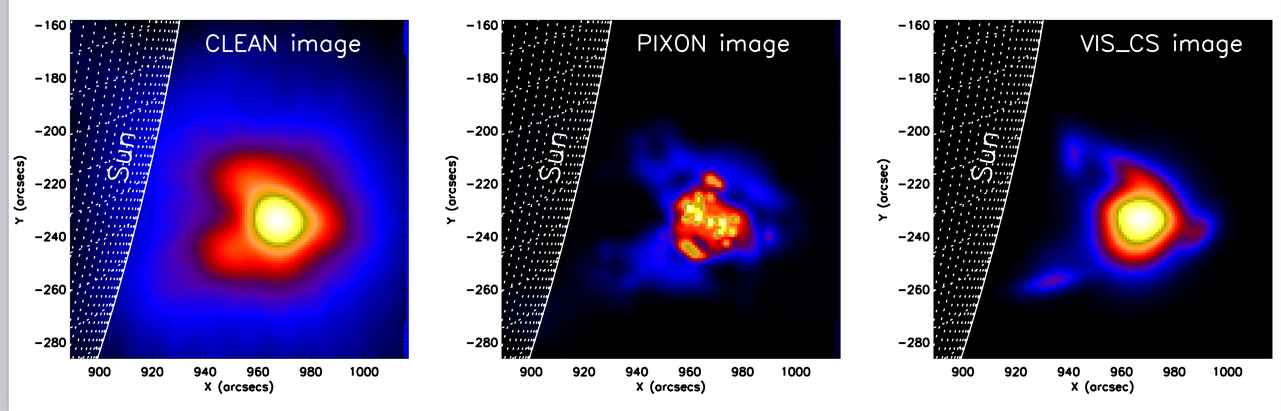
Tests with synthetic data

Compared to traditional image reconstruction algorithms, our algorithm shows above average resilience to noise, competitive reconstruction resolution and no need for parameter tuning.



Real-world data

We compare reconstructions of a real RHESSI solar flare observation (2012-09-19 05:21-05:22 UT) using three different algorithms: CLEAN has pronounced side-lobes and shows little source structure while PIXON tends to unphysically fragment the source. In contrast VIS_CS produces compact source sizes, little noise, and no fraying.



[1] Candès, Emmanuel J., Justin K. Romberg, and Terence Tao. "Stable signal recovery from incomplete and inaccurate measurements." Communications on pure and applied mathematics 59.8 (2006): 1207-1223.
[2] Gurobi Optimization, Inc. "Gurobi Linear Programming Solver 6.5" www.gurobi.com (2016)